1 Impact of a chalk

A piece of chalk strikes a wall at time $t = 0$. You are trying to determine how much time will elapse between the moment when the chalk hits the wall and the moment when it is pushed back at time t_c.

As you don't have the necessary knowledge to understand the propagation of waves within the chalk, you decide to use the Buckingham-Pi theorem to determine which variables will have the greatest impact on this time.

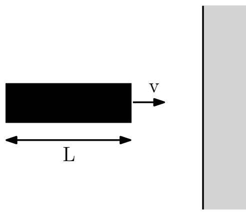

Figure 1: Projectile impacting a wall.

Question 1

What are the physical parameters that can affect this time? Write down the variable you are looking for (t_c) and the set of N independent variables.

Hint : The initial speed of the chalk is irrelevant. The parameters we are looking for should be related to the material and/or the dimensions of the chalk.

Question 2

Determine the dimensionally independent variables by building exponent matrix of the dimensions of $N + 1$ parameters and by determining the rank K of the matrix. How many dimensional groups do you need?

Question 3

Formulate the dimensionless parameter(s) of your system. Once this has been done, determine an equation defining the time t_c as a function of the other variables and the dimensionless parameter(s).

2 Fractional relationships

Consider a mode-I crack propagating in a brittle material. The critical stress intensity factor K_c determines when the crack propagates, and the critical strain energy release rate G_c determines the rate at which energy is transformed as a material undergoes fracture. You want to analyze the relationship between K_c, G_c and other physical properties of the material and geometry. The variables involved are:

- K_c : critical stress intensity factor $[\mathrm{Pa}\sqrt{\mathrm{m}}]$
- G_c : energy release rate $\left[\mathrm{J/m^2}\right]$
- E : Young's modulus
- σ_y : applied stress

- a : crack length

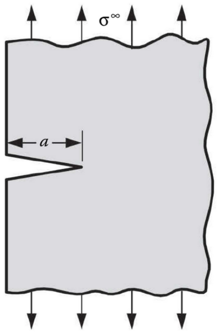

Figure 2: Mode-I fracture in a semi-infinite plate.

Question 1

Build the exponent matrix and determines its rank. How many dimensionless groups will you need?

Question 2

Formulates the dimensionless parameters of the system. Use K_c and G_c as starting points for the first two dimensionless parameters. What is the formulation of your system in terms of these dimensionless parameters?

Hint : one of your dimensionless parameter should be the strain.

Question 3

After careful experiments, you manage to find the following relation in plane stress conditions:

$$\pi_{K_c} = \sqrt{\pi_{G_c}}$$

From this relation, can you establish a relationship for G_C determined from the other variables in the system?

3 Transversal wave in a bar

A perturbation causes the leftward propagation of a wave in a bar of Young's modulus E and density ρ (see Figure 3). The bar is considered semi-infinite (infinite in $z = -\infty$ and clamped to a rigid wall at $z = 0$). Dissipative forces (such as friction, heat) are neglected.

Question 1

The total displacement field of the bar $u(z,t)$ can be given as the sum of the incident waves $u^I(z,t)$ coming from the perturbation and the reflected waves $u^R(z,t)$ at the wall. Which are the mechanical assumptions enabling us to decompose the total response of a system into the sum of several contributions ?

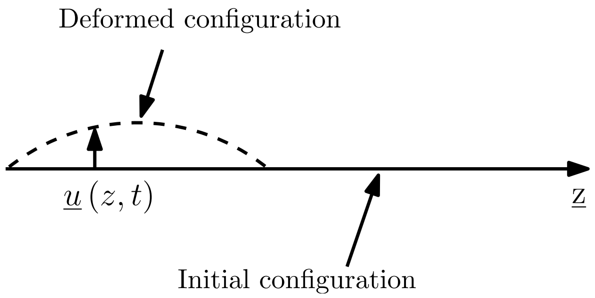

Figure 3: Wave traveling leftward in semi-infinite bar.

Question 2

Let us consider that the incident waves have an harmonic shape of the form:

$$u^I(z,t) = A\cos(k^I z - \omega^I t), \tag{1}$$

A being the amplitude of the wave. How can you connect k^I and ω^I ?
Hint : the wave speed v can also be written as $v = \frac{\partial z}{\partial t}$

Question 3

The reflected wave $u^R(z,t)$ can also be written as an harmonic wave

$$u^R(z,t) = B\cos(k^R z + \omega^R t). \tag{2}$$

Use the boundary condition at $z = 0$ to connect its shape with the one of the incident wave.

Question 4

When a incident harmonic wave interacts with its reflection, they form a stationary wave (phenomenon of resonance). Prove it for the considered 1d bar using the following trigonometric relationship

$$\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2}) \tag{3}$$

4 Additional : Firefighting hose

A firefighter is using a hose to direct water to extinguish a fire. The volumetric flow rate Q $\left[\mathrm{m}^3/\mathrm{s}\right]$ of the water depends on the pressure difference ΔP [Pa], the viscosity of water μ [Pa · s], the diameter of the hose D [m], and the length of the hose L [m]. The flow is assumed to be laminar.

You are trying to estimate how those parameters will influence the pressure loss, but you have no knowledge of fluid mechanics. You decide to apply a dimensional analysis.

Question 1

Determine the number of dimensionless parameters required, and formulate them.

Hint : The first two dimensionless parameters should be determined from Q and L.

Question 2

Doing some experiments, you figure out a single relation between the two dimensionless groups:

$$\pi_1 = f(\pi_2) = \frac{C}{\pi_2}$$

With π_1 the dimensionless parameter found with Q and π_2 the dimensionless parameter found with L. Knowing this relationship, how many experiences will you need to estimate the value of C, which is a constant?

Question 3

Thanks to your test, you can determine that $C = \frac{\pi}{128}$, where here π is the number pi. Determine a relationship that allows you to find ΔP as a function of the other parameters. Which physical law corresponds to this result?